



TITLE:

$\lambda$  Credibility (Information and mathematics of non-additivity and non-extensivity : contacts with nonlinearity and non-commutativity)

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## $\lambda$ Credibility

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### Abstract

This paper extends credibility measure in uncertainty theory to lambda credibility. Lambda credibility measure is a convex combination of possibility measure and necessity measure. We investigate differences between credibility measure and lambda credibility measure. Finally, we introduce lambda credibility expectation for a triangular fuzzy variable to have an insight on the differences between credibility and lambda credibility.

**Keywords:** uncertain theory, credibility measure, possibility measure, necessity measure, fuzzy variable, expectation operator

## 1 Introduction

Fuzzy set was first introduced by Zadeh [16] in 1965. This notion has been very useful in human decision making under uncertainty. We can see lots of papers which use this fuzzy set theory in K.Iwamura and B.Liu [1] [2], B.Liu and K.Iwamura [9] [10] [11], X.Ji and K.Iwamura [4], X.Gao and K.Iwamura [5], G.Wang and K.Iwamura [14], M.Wen and K.Iwamura [15] and others. We also have some books on fuzzy decision making under fuzzy environments such as D.Dubois and H.Prade [17], H-J.Zimmermann [18], M.Sakawa [20], J.Kacprzyk [19], B.Liu and A.O.Esogbue [8].

Recently B.Liu has founded a frequentist fuzzy theory with huge amount of applications in fuzzy mathematical programming. We see it in books such as B.Liu [3] [7] [6]. In his book [6] published in 2004, B.Liu [6] has succeeded in establishing an axiomatic foundation for uncertainty theory, where they have created a notion of credibility measure, which is a mean of possibility measure and necessity measure.

In this paper, we further define  $\lambda$  credibility measure which is a convex combination of possibility measure and necessity measure. Then we investigate differences between credibility measure and  $\lambda$  credibility measure. We will see that credibility measure is much better than  $\lambda$  credibility measure because the former can deal with many applications in mathematical programming in fuzzy environments. Yet, our  $\lambda$  credibility is also a special fuzzy measure in T.Murofushi and M.Sugeno [21]. Hence there could

come out some useful application of  $\lambda$ -credibility measure based on either Sugeno integral [13] or Choquet integral in the future, which is left unanswered in this paper.

The rest of the paper is organized as follows. The next section provides a brief review on the results of possibility, necessity and credibility. Section 3 presents the definition of  $\lambda$ -credibility and investigate its characteristics to recognize the importance of credibility measure. In section 4, we introduce  $\lambda$ -credibility expectation for a triangular fuzzy variable  $(a, b, c)$  to see that its formula differs depending on cases  $0 \leq a < b < c$ ,  $a < 0 \leq b < c$ ,  $a < b < 0 \leq c$ , while they all reduces to the same expression  $(a + 2b + c)/4$  if we set  $\lambda = 1/2$ . Finally in Section 5, we give a final conclusion on  $\lambda$  credibility.

## 2 Possibility, Necessity and Credibility

We start with the axiomatic definition of possibility given by B. Liu [6] in 2004. Let  $\Theta$  be an arbitrary nonempty set, and let  $\mathcal{P}(\Theta)$  be the power set of  $\Theta$ .

The four axioms are listed as follows:

**Axiom 1.**  $\text{Pos}\{\Theta\} = 1$ .

**Axiom 2.**  $\text{Pos}\{\emptyset\} = 0$ .

**Axiom 3.**  $\text{Pos}\{\cup_i A_i\} = \sup_i \text{Pos}\{A_i\}$  for any collection  $\{A_i\}$  in  $\mathcal{P}(\Theta)$ .

**Axiom 4.** Let  $\Theta_i$  be nonempty sets on which  $\text{Pos}_i\{\cdot\}$  satisfy the first three axioms,  $i = 1, 2, \dots, n$ , respectively, and  $\Theta = \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$ . Then

$$\text{Pos}\{A\} = \sup_{(\theta_1, \theta_2, \dots, \theta_n) \in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \dots \wedge \text{Pos}_n\{\theta_n\} \quad (1)$$

for each  $A \in \mathcal{P}(\Theta)$ . In that case we write  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \dots \wedge \text{Pos}_n$ .

We call  $\text{Pos}$  a possibility measure if it satisfies the first three axioms. We call the triplet  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  a possibility space.

**Note 2.1 :** We have  $\Theta_1, \text{Pos}_1$  and  $\Theta_2, \text{Pos}_2$  which satisfy the first three axioms and  $\Theta = \Theta_1 \times \Theta_2$ ,  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2$  which satisfy the four axioms.

**Theorem 2.1** (B. Liu, 2004). Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space. Then we have

- (a) 0-1 boundedness:  $0 \leq \text{Pos}\{A\} \leq 1$  for any  $A \in \mathcal{P}(\Theta)$ ;
- (b) monotonicity:  $\text{Pos}\{A\} \leq \text{Pos}\{B\}$  whenever  $A \subset B$ ;
- (c) subadditivity:  $\text{Pos}\{A \cup B\} \leq \text{Pos}\{A\} + \text{Pos}\{B\}$  for any  $A, B \in \mathcal{P}(\Theta)$ .
- (d)  $\text{Pos}\{A \cup B\} = \max(\text{Pos}\{A\}, \text{Pos}\{B\})$  for any  $A, B \in \mathcal{P}(\Theta)$ .

**Note 2.2 :** We see that

$$\text{Pos}\{A \cup B\} = \max(\text{Pos}\{A\}, \text{Pos}\{B\}) \quad (2)$$

naturally brings subadditivity in Theorem 2.1(c).

**Theorem 2.2** (B. Liu, 2004). Let  $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i), i = 1, 2, \dots, n$  be possibility spaces,  $\Theta = \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$  and  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \dots \wedge \text{Pos}_n$ . Then  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  is a possibility space, which is called *n-fold product possibility space* of  $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$  ( $i = 1, 2, \dots, n$ ).

**Theorem 2.3** (B. Liu, 2004). Let  $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i), i = 1, 2, \dots$  be possibility spaces. If  $\Theta = \Theta_1 \times \Theta_2 \times \dots$  and  $\text{Pos} = \text{Pos}_1 \wedge \text{Pos}_2 \wedge \dots$ , then the set function  $\text{Pos}$  is a possibility measure on  $\mathcal{P}(\Theta)$ , and  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  is a possibility space, which is called *infinite product possibility space* of  $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$  ( $i = 1, 2, \dots$ ).

Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space, and  $A$  a set in  $\mathcal{P}(\Theta)$ . Then the necessity measure of  $A$  is defined by

$$\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\}. \quad (3)$$

We have

**Theorem 2.4** (B. Liu, 2004). Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space. Then we have

- (a)  $\text{Nec}\{\Theta\} = 1, \text{Nec}\{\emptyset\} = 0$ ;
- (b) *0-1 boundedness*:  $0 \leq \text{Nec}\{A\} \leq 1$  for any  $A \in \mathcal{P}(\Theta)$ ;
- (c) *monotonicity*:  $\text{Nec}\{A\} \leq \text{Nec}\{B\}$  whenever  $A \subset B$ ;
- (d)  $\text{Nec}\{A\} + \text{Pos}\{A^c\} = 1$  for any  $A \in \mathcal{P}(\Theta)$ ;
- (e)  $\text{Nec}\{A \cap B\} = \min(\text{Nec}\{A\}, \text{Nec}\{B\})$  for any  $A, B \in \mathcal{P}(\Theta)$ ;
- (f)  $\text{Nec}\{A\} = 0$  whenever  $\text{Pos}\{A\} < 1$ ;

For a possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  and  $A$  a set in  $\mathcal{P}(\Theta)$ , B. Liu and Y-K. Liu [12] [6] have introduced the credibility measure of  $A$  by

$$\text{Cr}\{A\} = \frac{1}{2} (\text{Pos}\{A\} + \text{Nec}\{A\}). \quad (4)$$

We easily have

**Theorem 2.5** (B. Liu, 2004).

$$\text{Pos}\{A\} \geq \text{Cr}\{A\} \geq \text{Nec}\{A\} \quad (5)$$

for any  $A \in \mathcal{P}(\Theta)$  of a possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ .

**Theorem 2.6** (B. Liu, 2004). Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space. Then we have

- (a)  $\text{Cr}\{\Theta\} = 1, \text{Cr}\{\emptyset\} = 0$ ;
- (b) *0-1 boundedness*:  $0 \leq \text{Cr}\{A\} \leq 1$  for any  $A \in \mathcal{P}(\Theta)$ ;
- (c) *monotonicity*:  $\text{Cr}\{A\} \leq \text{Cr}\{B\}$  whenever  $A \subset B$ ;
- (d) *self duality*:  $\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1$  for any  $A \in \mathcal{P}(\Theta)$ ;
- (e) *subadditivity*:  $\text{Cr}\{A \cup B\} \leq \text{Cr}\{A\} + \text{Cr}\{B\}$  for any  $A, B \in \mathcal{P}(\Theta)$ .

We further have

**Theorem 2.7** *Let  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  be a possibility space. Let  $\text{Cr}\{\cdot\}$  be the credibility measure on  $\mathcal{P}(\Theta)$ . Then*

- (a)  $\text{Pos}\{A\} < 1$  implies  $\text{Cr}\{A\} = \frac{1}{2}\text{Pos}\{A\}$ ;
- (b)  $\text{Cr}\{A\} \geq \frac{1}{2}$  implies  $\text{Pos}\{A\} = 1$ .

A fuzzy variable is defined as a function from  $\Theta$  of a possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$  to the set of reals. Let  $\xi$  be a fuzzy variable defined on the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ . Then the set

$$\xi_\alpha = \{\xi(\theta) \mid \theta \in \Theta, \text{Pos}\{\theta\} \geq \alpha\}$$

is called the  $\alpha$ -level set of  $\xi$ . The set

$$\Theta^+ = \{\theta \in \Theta \mid \text{Pos}\{\theta\} > 0\}$$

is called the kernel of the possibility space and the set

$$\{\xi(\theta) \mid \theta \in \Theta, \text{Pos}\{\theta\} > 0\} = \{\xi(\theta) \mid \theta \in \Theta^+\}$$

is called the support of  $\xi$ .

Let  $\xi$  be a fuzzy variable defined on the possibility space  $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ . Then its membership function is derived through the possibility measure  $\text{Pos}$  by

$$\mu(x) = \text{Pos}\{\theta \in \Theta \mid \xi(\theta) = x\}, \quad x \in \mathbb{R}. \quad (6)$$

**Theorem 2.8** (B. Liu, 2004). *Let  $\mu : \mathbb{R} \rightarrow [0, 1]$  be a function with  $\sup \mu(x) = 1$ . Then there is a fuzzy variable whose membership function is  $\mu$ .*

### 3 $\lambda$ Credibility

Let  $\text{Pos}\{\cdot\}$  and  $\text{Nec}\{\cdot\}$  be a possibility measure and the necessity measure on  $\mathcal{P}(\Theta)$ . Let  $\lambda$  be a real number between 0 and 1, i.e.,  $\lambda \in [0, 1]$ . Define a set function  $\lambda\text{Cr}\{\cdot\}$  on  $\mathcal{P}(\Theta)$  by

$$\lambda\text{Cr}\{A\} = \lambda\text{Pos}\{A\} + (1 - \lambda)\text{Nec}\{A\}. \quad (7)$$

For  $\lambda = 1$ , we have  $\lambda\text{Cr}\{A\} = \text{Pos}\{A\}$  and for  $\lambda = 0$ , we have  $\lambda\text{Cr}\{A\} = \text{Nec}\{A\}$  and we see that  $0.5\text{Cr}\{A\} = \text{Cr}\{A\}$ , for any  $A \in \mathcal{P}(\Theta)$ .  $\lambda\text{Cr}\{\cdot\}$  is not  $\text{Cr}\{\cdot\}$  multiplied by  $\lambda$ !

We easily have

**Theorem 3.1** *Let  $\lambda$  be a real number such that  $0 \leq \lambda \leq 1$ . Then*

- (a)  $\lambda\text{Cr}\{\Theta\} = 1, \lambda\text{Cr}\{\emptyset\} = 0$ ;
- (b)  $\text{Nec}\{A\} \leq \lambda\text{Cr}\{A\} \leq \text{Pos}\{A\}$  for any  $A \in \mathcal{P}(\Theta)$  and so 0-1 boundedness holds for  $\lambda\text{Cr}\{\cdot\}$ ;

- (c) *monotonicity*:  $\lambda\text{Cr}\{A\} \leq \lambda\text{Cr}\{B\}$  whenever  $A \subset B$ ;  
 (d)  $2 \geq \lambda\text{Cr}\{A\} + \lambda\text{Cr}\{A^c\} \geq 1$  whenever  $\lambda \geq \frac{1}{2}$   
 while  $0 \leq \lambda\text{Cr}\{A\} + \lambda\text{Cr}\{A^c\} \leq 1$  whenever  $\lambda \leq \frac{1}{2}$  for any  $A \in \mathcal{P}(\Theta)$ ;  
 (e) *restricted subadditivity*:  $\lambda\text{Cr}\{A \cup B\} \leq \lambda\text{Cr}\{A\} + \lambda\text{Cr}\{B\}$  for any  $A, B \in \mathcal{P}(\Theta)$  provided that  $\lambda \geq \frac{1}{2}$ .

**Corollary 3.1** Let  $\lambda$  be such that  $0 < \lambda < 1$ . Then we get

- (a)  $\lambda\text{Cr}\{A\} = 1$  if and only if  $\text{Pos}\{A\} = \text{Nec}\{A\} = 1$ ,  
 (b)  $\lambda\text{Cr}\{A\} = 0$  if and only if  $\text{Pos}\{A\} = \text{Nec}\{A\} = 0$ .

**Theorem 3.2** Let  $0 \leq \lambda \leq 1$ . Suppose that there exists a set  $A \in \mathcal{P}(\Theta)$  such that  $\text{Pos}\{A\} > \text{Nec}\{A\}$  with  $\lambda\text{Cr}\{A\} + \lambda\text{Cr}\{A^c\} = 1$ . Then we get  $\lambda = \frac{1}{2}$ .

**Corollary 3.2** Suppose that there exist  $s, t$  with  $0 \leq s < t \leq 1$  such that for any  $\lambda \in [s, t]$ , any  $A \in \mathcal{P}(\Theta)$ ,  $\lambda\text{Cr}\{A\} + \lambda\text{Cr}\{A^c\} = 1$ . Then we have  $\text{Pos}\{A\} = \text{Nec}\{A\}$  for any  $A \in \mathcal{P}(\Theta)$ .

**Example 3.1.** Let  $\Theta = \{\theta_1, \theta_2\}$  with  $\text{Pos}\{\theta_1\} = 1.0, \text{Pos}\{\theta_2\} = 0.8$ . Then  $\text{Nec}\{\theta_1\} = 1 - \text{Pos}\{\theta_2\} = 0.2, \text{Nec}\{\theta_2\} = 1 - 1 = 0$ .  $\lambda\text{Cr}\{\Theta\} = 1, \lambda\text{Cr}\{\emptyset\} = 0, \lambda\text{Cr}\{\theta_1\} = 0.8\lambda + 0.2, \lambda\text{Cr}\{\theta_2\} = 0.8\lambda, \lambda\text{Cr}\{\theta_1\} + \lambda\text{Cr}\{\theta_2\} = 1.6\lambda + 0.2$  for  $0 \leq \lambda \leq 1$ . Therefore

$$\lambda\text{Cr}\{\theta_1\} + \lambda\text{Cr}\{\theta_2\} = 1 \quad \text{if and only if} \quad \lambda = \frac{1}{2}. \quad (8)$$

For  $\lambda = 0.4, \lambda\text{Cr}\{\theta_1\} + \lambda\text{Cr}\{\theta_2\} = 0.84 < 1$ .

And for  $\lambda = 0.6, \lambda\text{Cr}\{\theta_1\} + \lambda\text{Cr}\{\theta_2\} = 1.16 > 1$ .

So,  $\lambda\text{Cr}\{\cdot\}$  is not self dual for any  $\lambda, 0 < \lambda < 1$  with  $\lambda \neq \frac{1}{2}$ .

**Example 3.2.** Let  $\xi$  be a triangular fuzzy variable with its membership function

$$\mu(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 1 \\ -x + 2, & \text{if } 1 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Then we get

$$\text{Pos}\{\xi \leq x\} = \begin{cases} 0, & \text{if } x \leq 0 \\ x, & \text{if } 0 < x \leq 1 \\ 1, & \text{if } x > 1. \end{cases}$$

and

$$\text{Pos}\{\xi > x\} = \begin{cases} 1, & \text{if } x \leq 1 \\ -x + 2, & \text{if } 1 < x \leq 2 \\ 0, & \text{if } x > 2. \end{cases}$$

For  $0 < x < 1, \text{Pos}\{\xi \leq x\} + \text{Pos}\{\xi > x\} = x + 1 > 1$ . Therefore, we get  $\lambda = \frac{1}{2}$  provided that  $\lambda\text{Cr}\{\cdot\}$  is self dual. This fact suggests the importance of  $0.5\text{Cr}\{\cdot\}$ , i.e., credibility measure. Furthermore, we have

**Theorem 3.3** Let  $\xi$  be a triangular fuzzy variable with its membership function

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ \frac{x-c}{b-c}, & \text{if } b < x \leq c \\ 0, & \text{otherwise} \end{cases}$$

where  $a \leq b \leq c$ . Suppose that  $\text{Pos}\{\xi \leq x\} + \text{Pos}\{\xi > x\} = 1$  holds for any real number  $x$ , i.e.,  $\lambda\text{Cr}\{\cdot\}$  is self dual for fuzzy events  $\{\xi \leq x\}$ . Then we have  $a = b = c$ .

**Example 3.3:** Let  $\xi$  be a fuzzy variable with its membership function  $\mu(x) = \delta_a(x)$ , where

$$\delta_a(x) = \begin{cases} 1, & x = a \\ 0, & x \neq a. \end{cases}$$

We call  $\delta_a$  a Dirac measure. Then we get

$$\text{Pos}\{\xi \leq x\} = \begin{cases} 0, & x < a \\ 1, & x \geq a \end{cases} \quad \text{and} \quad \text{Pos}\{\xi > x\} = \begin{cases} 1, & x < a \\ 0, & x \geq a. \end{cases}$$

Hence we get  $\text{Pos}\{\xi \leq x\} + \text{Pos}\{\xi > x\} = 1$  for any number  $x$ . Therefore  $\lambda\text{Cr}\{\xi \leq x\} + \lambda\text{Cr}\{\xi > x\} = 1$  for any number  $x$ .  $\lambda\text{Cr}\{\cdot\}$  is self dual for fuzzy events  $\{\xi \leq x\}$ , where  $x$  is a real number.

## 4 $\lambda$ -Credibility Expectation

Let  $\xi$  be a triangular fuzzy variable  $(a, b, c)$  with its membership function  $\mu(x)$  as follows;

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ \frac{x-c}{b-c}, & \text{if } b < x \leq c \\ 0, & \text{otherwise.} \end{cases}$$

Then we get

$$\lambda\text{Cr}\{\xi \leq x\} = \begin{cases} 1, & \text{if } x \leq a \\ \frac{b-\lambda a}{b-a} - \frac{1-\lambda}{b-a}x, & \text{if } a < x \leq b \\ -\frac{\lambda c}{b-c} + \frac{\lambda}{b-c}x, & \text{if } b < x \leq c \\ 0, & \text{if } c < x. \end{cases}$$

Define  $\lambda$ -credibility expectation  $\lambda\text{Cr}\{\cdot\}$  of  $\xi$  by

$$E_{\lambda\text{Cr}}[\xi] = \int_0^{+\infty} \lambda\text{Cr}\{\xi \geq x\}dx - \int_{-\infty}^0 \lambda\text{Cr}\{\xi \leq x\}dx,$$

where integrals are defined through Lebesgue integral, i.e., well defined in case either of the two takes finite value.

Then we see the followings,

in case  $0 \leq a < b < c$ , we get

$$\begin{aligned}
 E_{\lambda Cr}[\xi] &= \int_0^{+\infty} \lambda Cr\{\xi \geq x\} dx \\
 &= \int_0^a 1 dx + \int_a^b \left\{ \frac{b-\lambda a}{b-a} - \frac{1-\lambda}{b-a} x \right\} dx \\
 &= \frac{b+a+(c-a)\lambda}{2},
 \end{aligned} \tag{9}$$

in case  $a < b < 0 \leq c$ , we get

$$E_{\lambda Cr}[\xi] = \left( \frac{-c^2}{2(b-c)} + \frac{a-b}{2} + \frac{b^2-2bc}{2(b-c)} \right) \lambda + \frac{b^2}{2(b-c)}. \tag{10}$$

If we set  $\lambda = \frac{1}{2}$  in equation (9),(10) we get  $E_{0.5Cr}[\xi] = \frac{a+2b+c}{4}$ , which is identical to the credibility expectation for a triangular fuzzy variable  $\xi$ . This fact shows us the importance and adequateness of the credibility expectation operator. Finally let us see the case of  $a=0, b=1, c=3$ . In this case we get

$$E_{\lambda Cr}[\xi] = \frac{1}{2} + \frac{3}{2}\lambda,$$

which coincides with the credibility expectation of  $\xi$ ,  $E[\xi] = \frac{0+2 \cdot 1+3}{4} = \frac{5}{4}$ , if we take  $\lambda = \frac{1}{2}$ .

We further see that  $\frac{1}{2} \leq E_{\lambda Cr}[\xi] \leq 2$  and  $\frac{1}{2}(\frac{1}{2}+2) = \frac{5}{4} = E[\xi]$ , where  $E_{Nec}[\xi] = \frac{1}{2}$  and  $E_{Pos}[\xi] = 2$ .

## 5 Conclusion

We have introduced  $\lambda$  credibility  $\lambda Cr\{ \}$  on a possibility space  $(\Theta, \mathcal{P}(\Theta), Pos)$  and have shown that  $\lambda Cr\{ \}$  is 0-1 bounded, monotone for set inclusion, over self dual or under self dual according to  $\lambda \geq \frac{1}{2}$  or  $\lambda \leq \frac{1}{2}$  and that it satisfies restricted subadditivity. We further have shown that for a non-trivial possibility space, where there exists a set  $A \in \mathcal{P}(\Theta)$  such that  $Pos\{A\} > Nec\{A\}$ , self duality of  $\lambda Cr\{ \}$  naturally leads to  $\lambda = \frac{1}{2}$ . We have given some examples for which self duality naturally holds when  $\lambda = \frac{1}{2}$ , or a triangular fuzzy variable  $\xi$  reduces to a Dirac measure if we demand  $\lambda Cr\{ \}$  be self dual for fuzzy events  $\{\xi \leq x\}$ . Finally we have defined  $\lambda Cr\{ \}$  expectation of a fuzzy variable  $\xi$ . For a triangular fuzzy variable  $(a, b, c)$ , the  $\lambda Cr\{ \}$  expectation  $E_{\lambda Cr}[\xi]$  has different formula in  $\lambda$ , dependent on  $0 < a$  or  $a < 0 < b$  or  $a < b < 0 < c$ . But if we let  $\lambda = \frac{1}{2}$  then they all reduce to  $(a+2b+c)/4$ , the expectation of  $\xi$  in the



sense of B.Liu [6] in 2004. So, we believe that  $0.5Cr\{ \} = Cr\{ \}$  is the most useful one among  $\lambda Cr\{ \}$ ,  $0 < \lambda < 1$ .

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